Sirindhorn International Institute of Technology

Thammasat University

School of Information, Computer and Communication Technology

ECS332 2012/1 Part II.2 Dr.Prapun

5 Quadrature Amplitude Modulation (QAM)

Definition 5.1. One of the possible definition for the ***bandwidth*** (***BW***) of a signal is the difference between the highest frequency and the lowest frequency in the positive-f part of the signal spectrum.

Example 5.2.

5.3. Rough Approximation: If $g_1(t)$ and $g_2(t)$ have bandwidths B_1 and B_2 Hz, respectively, the bandwidth of $g_1(t)g_2(t)$ is $B_1 + B_2$ Hz.

This result follows from the application of the width property[7] of convolution[8] to the convolution-in-frequency property.

Consequently, if the bandwidth of $g(t)$ is B Hz, then the bandwidth of $g^2(t)$ is $2B$ Hz, and the bandwidth of $g^n(t)$ is nB Hz. We mentioned this property in 2.26.

5.4. BW Inefficiency in DSB-SC: Recall that for real-valued baseband signal $m(t)$, the conjugate symmetry property from 2.16 says that

$$M(-f) = (M(f))^*.$$

[7]This property states that the width of $x * y$ is the sum of the widths of x and y.

[8]The width property of convolution does not hold in some pathological cases. See [3, p 98].

Solution: SSB
QAM ← our focus

The DSB spectrum has two sidebands: the upper sideband (USB) and the lower sideband (LSB), both containing complete information about the baseband signal $m(t)$. As a result, DSB signals occupy twice the bandwidth required for the baseband. To improve the spectral efficiency of amplitude modulation, there exist two basic schemes to either utilize or remove the spectral redundancy:

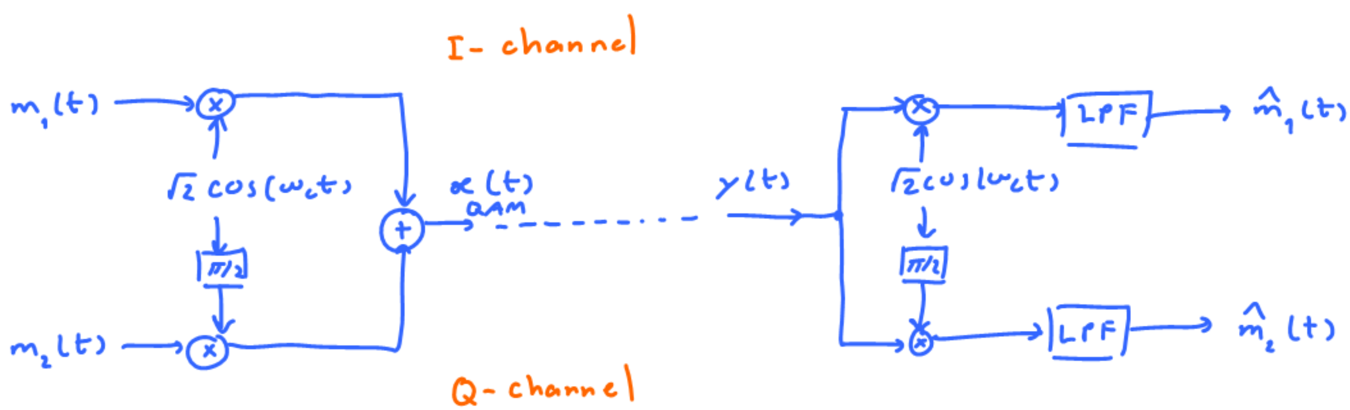
(a) Single-sideband (SSB) modulation, which removes either the LSB or the USB so that for one message signal $m(t)$, there is only a bandwidth of B Hz.

(b) Quadrature amplitude modulation (QAM), which utilizes spectral redundancy by sending two messages over the same bandwidth of $2B$ Hz.

We will only discussed QAM here. SSB discussion can be found in [2, Sec 4.4], [9, Section 3.1.3] and [3, Section 4.5].

Definition 5.5. In ***quadrature amplitude modulation*** (***QAM***) or ***quadrature multiplexing***, two baseband real-valued signals $m_1(t)$ and $m_2(t)$ are transmitted simultaneously via the corresponding QAM signal:

$$x_{\text{QAM}}(t) = m_1(t)\sqrt{2}\cos(\omega_c t) + m_2(t)\sqrt{2}\sin(\omega_c t).$$



- QAM operates by transmitting two DSB signals via carriers of the same frequency but in phase quadrature.
- QAM can be exactly generated without requiring sharp cutoff bandpass filters.

- Both modulated signals simultaneously occupy the same frequency band.
- The upper channel is also known as the ***in-phase*** (***I***) channel and the lower channel is the ***quadrature*** (***Q***) channel.

5.6. ***Demodulation***: The two baseband signals can be separated at the receiver by synchronous detection:

$m_1(t)\sqrt{2}\cos(\omega_c t) + m_2(t)\sqrt{2}\sin(\omega_c t)$

$$\mathrm{LPF}\left\{x_{\mathrm{QAM}}(t)\sqrt{2}\cos(\omega_c t)\right\} = m_1(t) \leftarrow$$

$$\mathrm{LPF}\left\{x_{\mathrm{QAM}}(t)\sqrt{2}\sin(\omega_c t)\right\} = m_2(t) \leftarrow \text{HW}$$

$x_{QAM}(t)\sqrt{2}\cos = m_1(t)\, 2\cos^2(\omega_c t) + \underbrace{m_2(t)\, 2\cos(\omega_c t)\sin(\omega_c t)}_{=\; m_2(t)\sin(2\omega_c t)}$

$= m_1(t) + m_1(t)\cos(2\omega_c t) + m_2(t)\sin(2\omega_c t)$

freq. content around $2f_c$ → won't pass the LPF

$x_{QAM}(t)\sqrt{2}\sin(\omega_c t) = \cdots \leftarrow$ HW

- $m_1(t)$ and $m_2(t)$ can be separately demodulated.

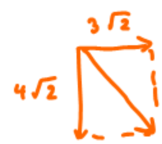
5.7. Sinusoidal form: $x_{QAM}(t) = m_1(t)\sqrt{2}\cos(\omega_c t) + m_2(t)\sqrt{2}\sin(\omega_c t)$

$$x_{\mathrm{QAM}}(t) = \sqrt{2}E(t)\cos(2\pi f_c t + \theta(t)),$$

Ex. $m_1 = 3$, $m_2 = 4$

$x_{QAM}(t) = 3\sqrt{2}\cos + \underbrace{4\sqrt{2}\sin}_{4\sqrt{2}\cos(\omega_c t - 90^\circ)}$

$3\sqrt{2}\angle 0^\circ + 4\sqrt{2}\angle -90^\circ$



$= 5\sqrt{2}\angle -53^\circ$

$= 5\sqrt{2}\cos(\omega_c t - 53^\circ)$

where

$$E(t) = \sqrt{m_1^2(t) + m_2^2(t)}$$

$$\theta(t) = -\tan^{-1}\left(\frac{m_2(t)}{m_1(t)}\right)$$

5.8. Complex form:

check this →

$$x_{\mathrm{QAM}}(t) = \sqrt{2}\mathrm{Re}\left\{(m(t))\, e^{j2\pi f_c t}\right\}$$

where $m(t) = m_1(t) - jm_2(t)$.

- If we use $-\sin(\omega_c t)$ instead of $\sin(\omega_c t)$,

$$x_{\text{QAM}}(t) = m_1(t)\sqrt{2}\cos(\omega_c t) - m_2(t)\sqrt{2}\sin(\omega_c t)$$

  and

$$m(t) = m_1(t) + jm_2(t).$$

- We refer to $m(t)$ as the ***complex envelope*** (or ***complex baseband signal***) and the signals $m_1(t)$ and $m_2(t)$ are known as the ***in-phase*** and ***quadrature(-phase)*** components of $x_{\text{QAM}}(t)$.

- The term "quadrature component" refers to the fact that it is in phase quadrature ($\pi/2$ out of phase) with respect to the in-phase component.

- Key equation:

$$\text{LPF}\left\{\underbrace{\left(\text{Re}\left\{m(t)\times\sqrt{2}e^{j2\pi f_c t}\right\}\right)}_{x(t)}\times\left(\sqrt{2}e^{-j2\pi f_c t}\right)\right\} = m(t).$$

5.9. Three equivalent ways of saying exactly the same thing:

(a) the complex-valued envelope $m(t)$ complex-modulates the complex carrier $e^{j2\pi f_c t}$,

(b) the real-valued amplitude $E(t)$ and phase $\theta(t)$ real-modulate the amplitude and phase of the real carrier $\cos(\omega_c t)$,

(c) the in-phase signal $m_1(t)$ and quadrature signal $m_2(t)$ real-modulate the real in-phase carrier $\cos(\omega_c t)$ and the real quadrature carrier $\sin(\omega_c t)$.

5.10. References: [2, p 164–166], [9, Sect. 2.9.4], [3, Sect. 4.4], and [6, Sect. 1.4.1]

5.11. Question: In engineering and applied science, measured signals are real. Why should real measurable effects be represented by complex signals?

Answer: One complex signal (or channel) can carry information about two real signals (or two real channels), and the algebra and geometry of analyzing these two real signals as if they were one complex signal brings economies and insights that would not otherwise emerge.

6 Amplitude modulation: AM

6.1. The analysis of DSB-SC in the earlier sections illustrates that the spectrum of a DSB signal does not contain a **discrete** spectral component at the carrier frequency unless $m(t)$ has a DC component. This is why we referred to it as a ***suppressed carrier*** system.

6.2. DSB-SC amplitude modulation is easy to understand and to analyze in both time and frequency domains. However, analytical simplicity is not always accompanied by an equivalent simplicity in practical implementation.

Problem: The (coherent) demodulation of DSB-SC signal requires the receiver to possess a carrier signal that is synchronized with the incoming carrier. This requirement is not easy to achieve in practice because the modulated signal may have traveled hundreds of miles and could even suffer from some unknown frequency shift.

6.3. If a carrier component is transmitted along with the DSB signal, demodulation can be simplified.

$x(t) = m(t)\cos(\omega_c t) + A\cos(\omega_c t) = (m(t)+A)\cos(\omega_c t)$

(a) The received carrier component can be extracted using a narrowband bandpass filter and can be used as the demodulation carrier. (There is no need to generate a carrier at the receiver.)

A is large enough

(b) If the carrier amplitude is sufficiently large, the need for generating a demodulation carrier can be completely avoided.

- This will be the focus of this section.

Definition 6.4. For AM, the transmitted signal is typically defined as

$$x_{\mathrm{AM}}(t) = (A + m(t))\cos(2\pi f_c t) = \underbrace{A\cos(2\pi f_c t)}_{\text{carrier}} + \underbrace{m(t)\cos(2\pi f_c t)}_{\text{sidebands}}$$

Assume A is large enough such that $A + m(t) \geq 0$

6.5. Trade-off:

(a) *Disadvantage*:

- Higher power and hence higher cost required at the transmitter
- The carrier component is wasted power as far as information transfer is concerned.
- This fact can completely preclude the use of AM in power-limited applications.

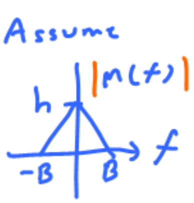
(b) *Advantage*:

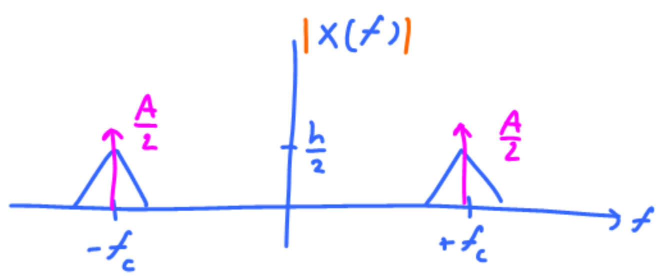
- Coherent reference is not needed for demodulation.
- Demodulator becomes simple and inexpensive.
- For broadcast system such as commercial radio (with a huge number of receivers for each transmitter,
  - any cost saving at the receiver is multiplied by the number of receiver units.
  - it is more economical to have one expensive high-power transmitter and simpler, less expensive receivers.

(c) Conclusion: Broadcasting systems tend to favor the trade-off by migrating cost from the (many) receivers to the (fewer) transmitters.

6.6. Spectrum of x_{AM}:

$$x_{AM}(t) = A\cos(\omega_c t) + \underbrace{m(t)\cos(\omega_c t)}$$





- Basically the same as that of DSB-SC except for the two additional impulses at $\pm f_c$.

Definition 6.7. Consider a signal $A(t)\cos(2\pi f_c t)$. If $A(t)$ varies slowly in comparison with the sinusoidal carrier $\cos(2\pi f_c t)$, then the ***envelope*** $E(t)$ of $A(t)\cos(2\pi f_c t)$ is $|A(t)|$.

6.8. *Envelope of AM signal*: See Figure 9. For AM signal, $A(t) = A + m(t)$.

(a) If $\forall t, A(t) > 0$, then $E(t) = A(t) = A + m(t)$

- The envelope has the same shape as $m(t)$.
- We can detect the desired signal $m(t)$ by detecting the envelope (envelope detection).

(b) If $\exists t, A(t) < 0$, then $E(t) \neq A(t)$.

- The envelope shape differs from the shape of $m(t)$ because the negative part of $A + m(t)$ is rectified.

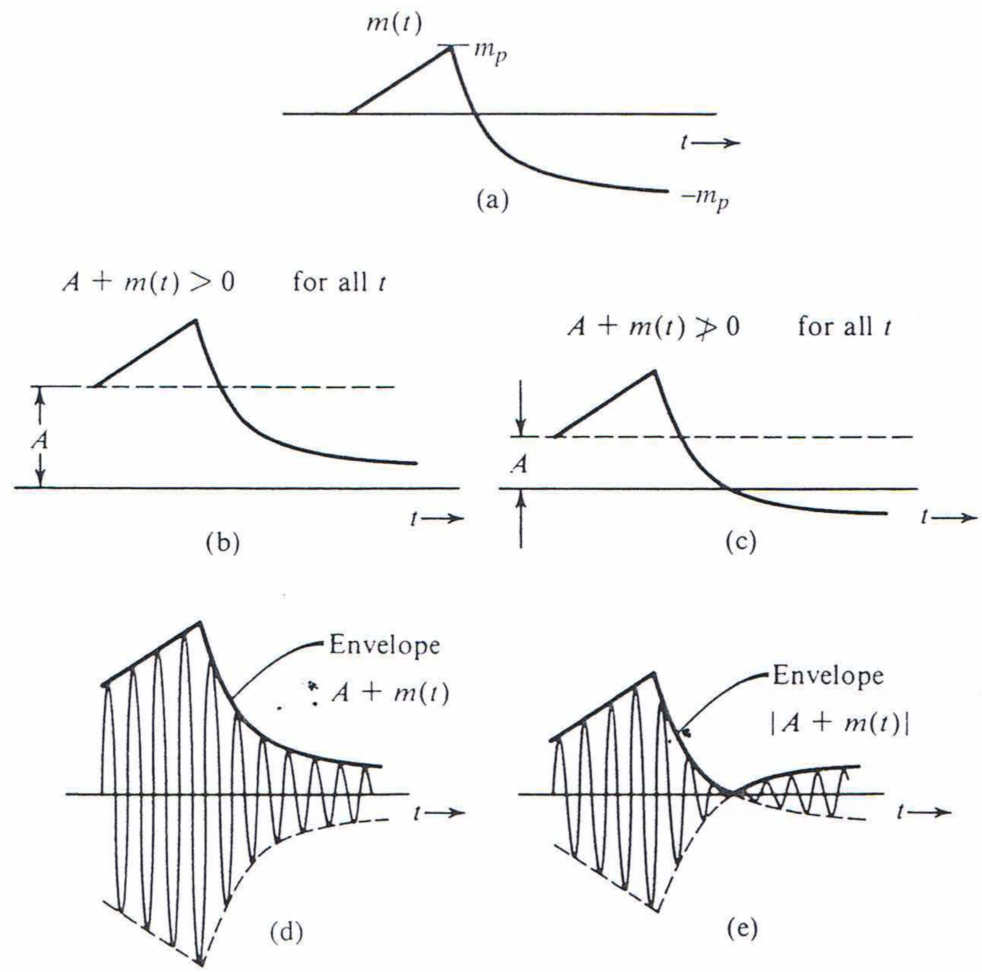


Figure 9: AM signal and its envelope [4, Fig 4.8]

6.9. Summary of AM Concept:

- The carrier term $A\cos(2\pi f_c t)$ is added.
- The size of A affects the time domain envelope of the modulated signal.
- A should be large enough to ensure that $A+m(t)$ is always nonnegative.
  - If $\forall t, m(t) \geq 0$, then there is no need to add any carrier. The DSB-SC signal can be detected by envelope detection.

6.10. ***Demodulation*** of AM Signals via rectifier detector: The receiver will first recover $A+m(t)$ and then remove A. Note that, conceptually, the received signal is the same as DSB-SC signal except that the $m(t)$ in the DSB-SC signal is replaced by $A(t) = A+m(t)$. We will also assume that A is large enough so that $A(t) \geq 0$.

Recall the key equation of ***switching demodulator*** (38):

$$\mathrm{LPF}\{A(t)\cos(2\pi f_c t) \times 1[\cos(2\pi f_c t) \geq 0]\} = \frac{1}{\pi}A(t) \tag{39}$$

We noted before that this technique requires the switching to be in sync with the incoming cosine.

When $\forall t, A(t) \geq 0$, we can replace the switching demodulator by the ***rectifier demodulator/detector***. In which case, we suppress the negative part of $m(t)\cos(\omega_c t)$ using a diode (half-wave rectifier). This is mathematically equivalent to switching demodulator in (38) and (39).

What is the effect of HWR

$x_{AM}(t) = \underbrace{(A+m(t))}_{\geq 0}\cos \to \boxed{HWR} \to (A+m(t))\cos \times 1[\cos] \xrightarrow{LPF} (A+m(t))\frac{1}{\pi}$

$$f_{HWR}(x) = \begin{cases} x & x \geq 0 \\ 0 & x < 0 \end{cases}$$

$$f_{HWR}(x_{AM}(t)) = f_{HWR}((A+m(t))\cos) = \begin{cases} (A+m(t))\cos(\omega_c t), & \overbrace{(\cancel{A+m(t)})}^{\geq 0}\cos\omega_c t \geq 0 \\ 0, & (\cancel{A+m(t)})\cos\omega_c t < 0 \end{cases}$$

$$= (A+m(t))\cos(\omega_c t) \times \underbrace{\begin{cases} 1, & \cos\omega_c t \geq 0 \\ 0, & \cos\omega_c t < 0 \end{cases}}$$

$g_{HWR}(t) = 1[\cos(\omega_c t) \geq 0]$ ← ON-OFF switching function.

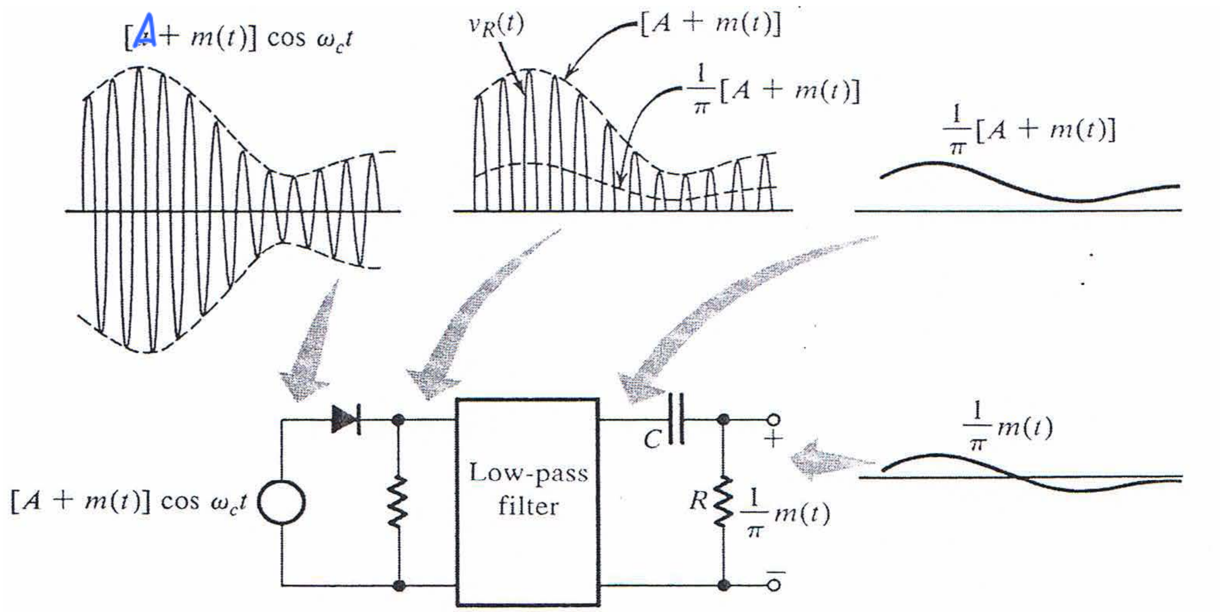


Figure 10: Rectifier detector for AM [4, Fig. 4.10].

- It is in effect synchronous detection performed without using a local carrier [3, p 167].
- This method needs $A(t) \geq 0$ so that the sign of $A(t)\cos(\omega_c t)$ will be the same as the sign of $\cos(\omega_c t)$.
- The dc term $\frac{A}{\pi}$ may be blocked by a capacitor to give the desired output $m(t)/\pi$.

6.11. Demodulation of AM signal via ***envelope detector***:

- Design criterion of RC:

$$2\pi B \ll \frac{1}{RC} \ll 2\pi f_c.$$

- The envelope detector output is $A + m(t)$ with a ripple of frequency f_c.
- The dc term can be blocked out by a capacitor or a simple RC high-pass filter.
- The ripple may be reduced further by another (low-pass) RC filter.

6.12. References: [2, p 198–199], [4, Section 4.3] and [9, Section 3.1.2].

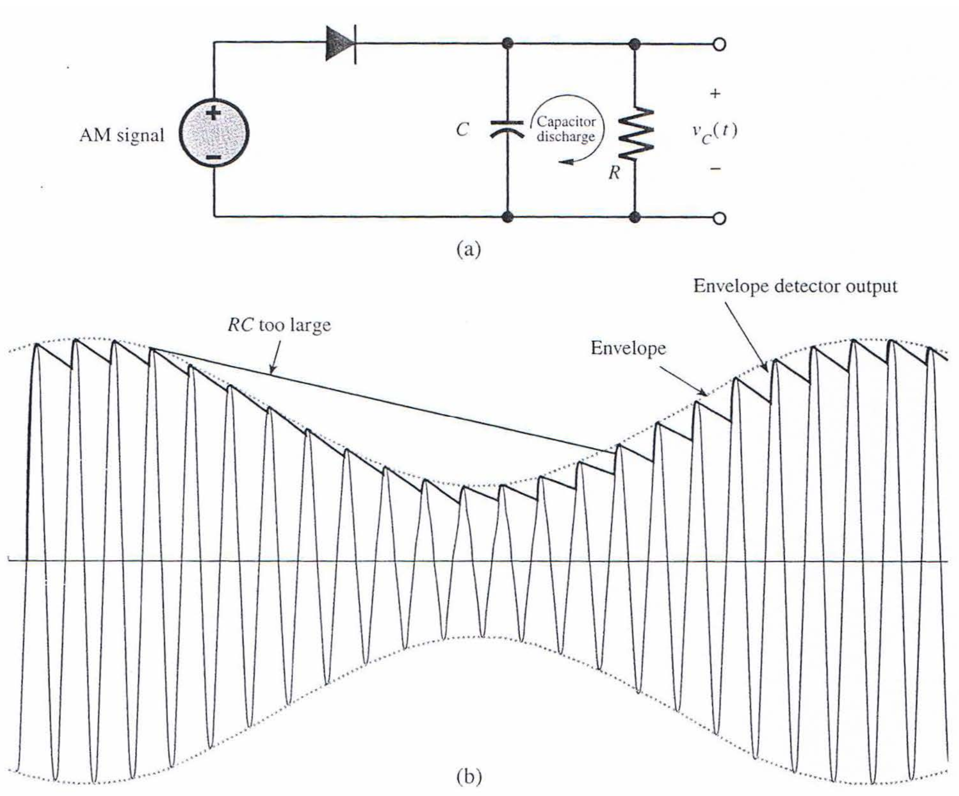


Figure 11: Envelope detector for AM [4, Fig. 4.11].